

2-Layer Fan-Planarity in Polynomial Time

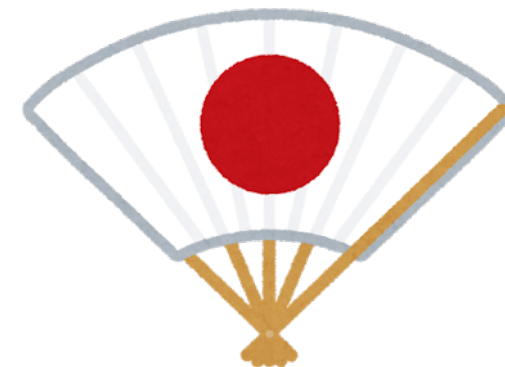
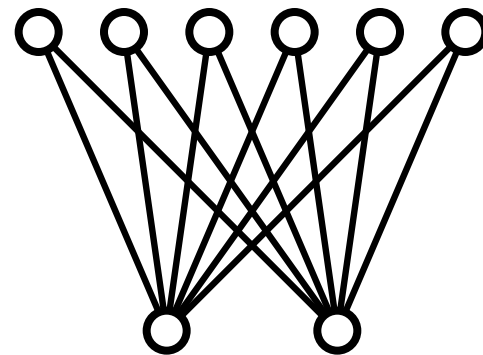
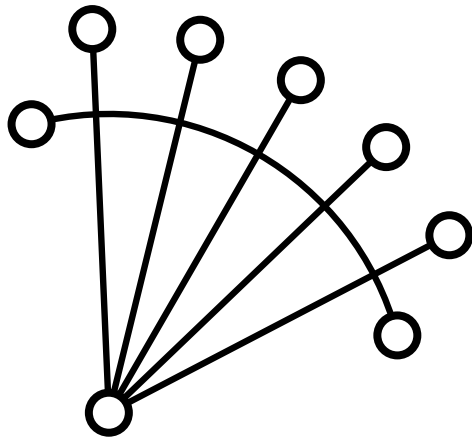
Yasuaki Kobayashi
(Hokkaido University)

Yuto Okada
(Nagoya University)

2-Layer **Fan-Planarity** in Polynomial Time

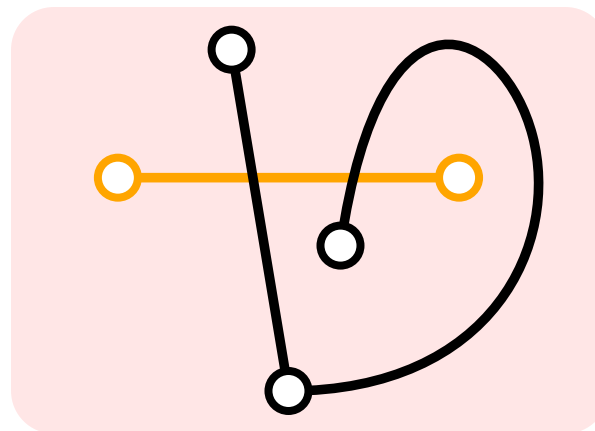
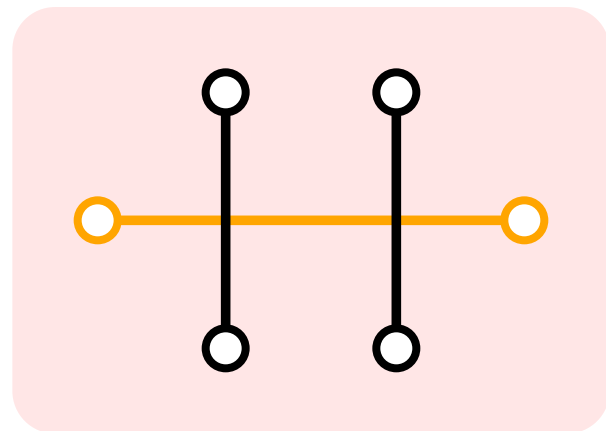
fan-planar drawing

Every edge e and the edges crossing e form a "**fan**".

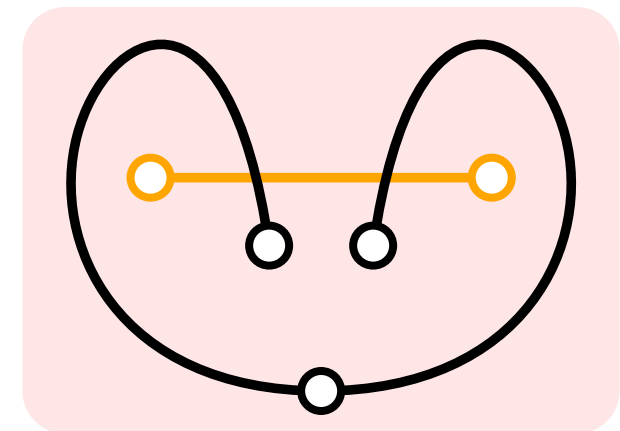


forbidden patterns

weakly fan-planar



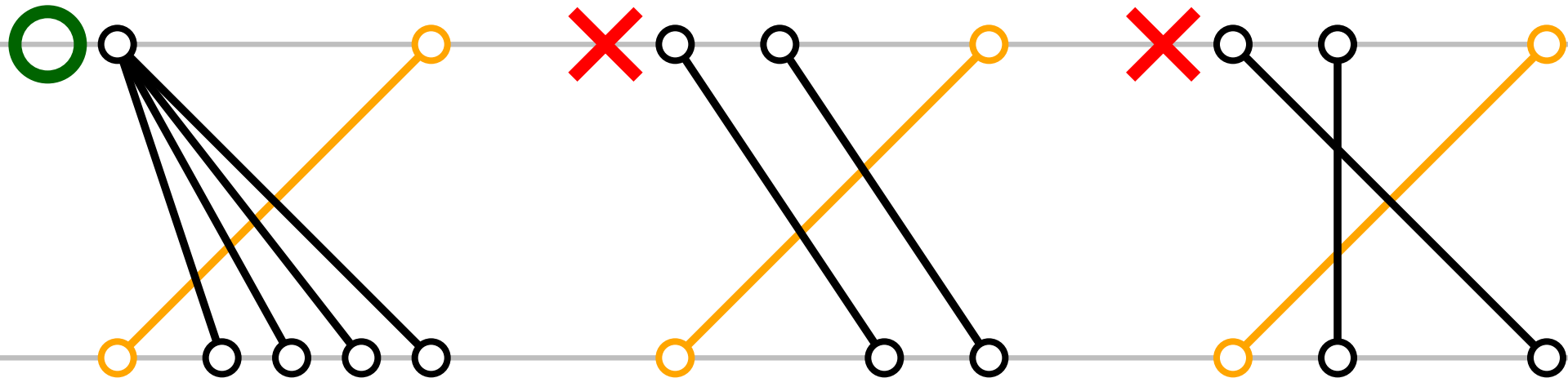
strongly fan-planar



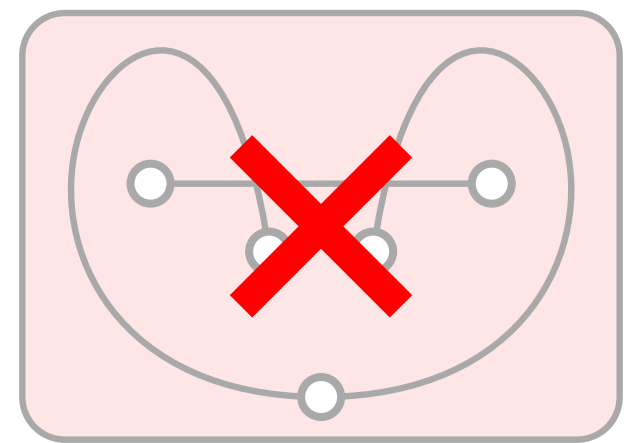
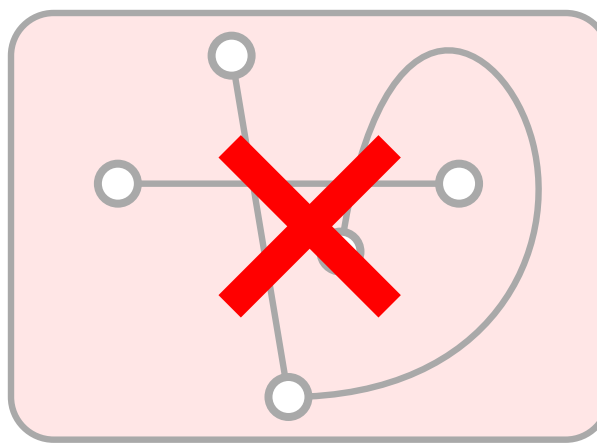
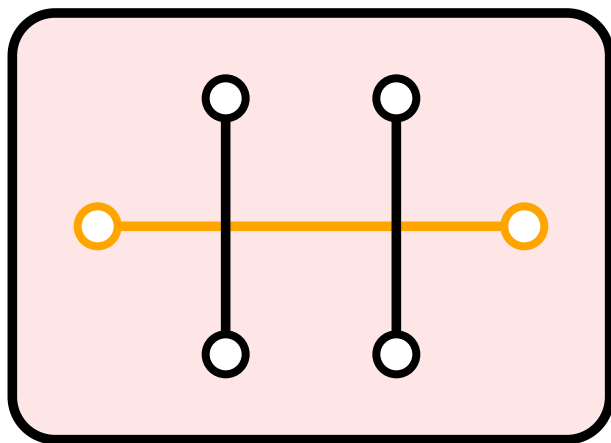
2-Layer Fan-Planarity in Polynomial Time

2-layer fan-planar drawing

For **every edge e** ,
the edges crossing e share a common endpoint.



forbidden patterns



2-Layer Fan-Planarity in Polynomial Time

2-LAYER FAN-PLANARITY

Given: a bipartite graph $G = (X \cup Y, E)$

Asks: does G admit a 2-layer fan-planar drawing?

Known Results

- linear-time for biconnected graphs

[BCDGKKMT, GD 2015 & JGAA (2017)]

- linear-time for trees

[BCKMNR, MFCS 2020]

Our Contribution

- **polynomial-time** for general graphs

$O(n^c)$ with $c \approx 55$

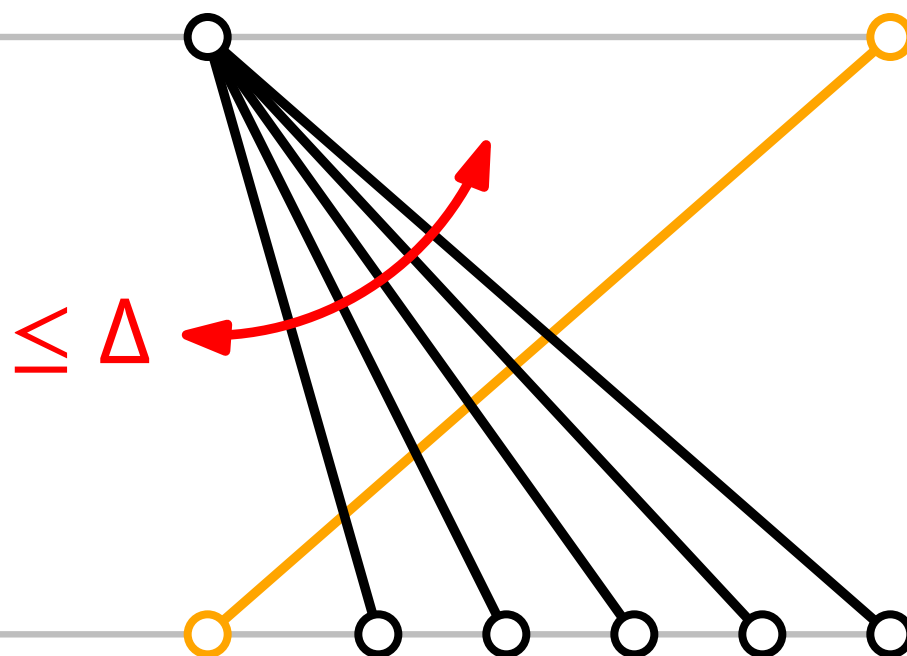


Key Observation

Observation

Let Δ be the maximum degree of a graph G .
Every 2-layer fan-planar drawing of G is Δ -planar.

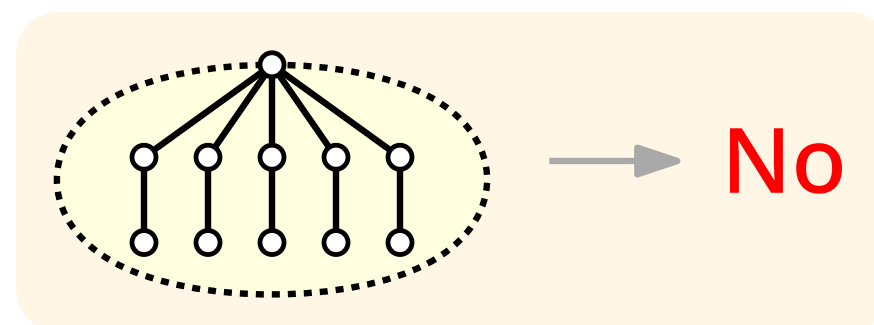
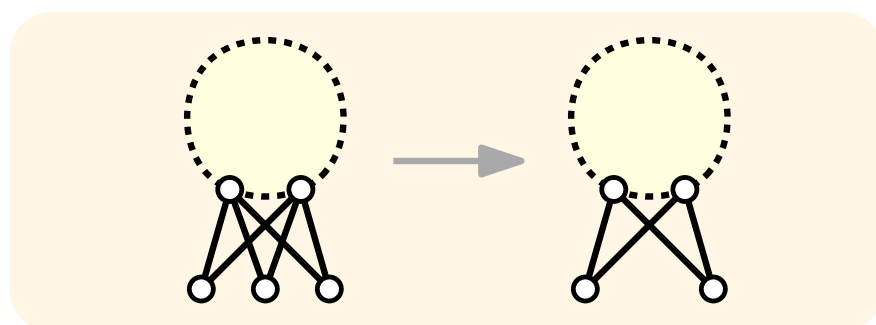
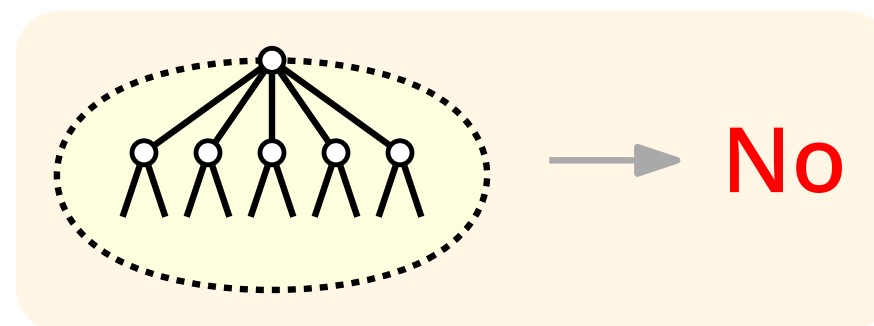
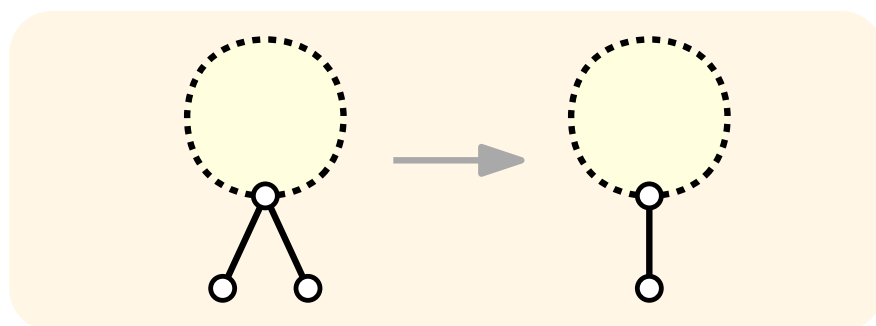
(k -planar: every edge involves at most k crossings.)



Our Approach

① Degree Reduction

Apply 4 rules exhaustively $\rightarrow \Delta(G) \leq 13$



② Search for a 2-layer fan-planar **13-planar** drawing

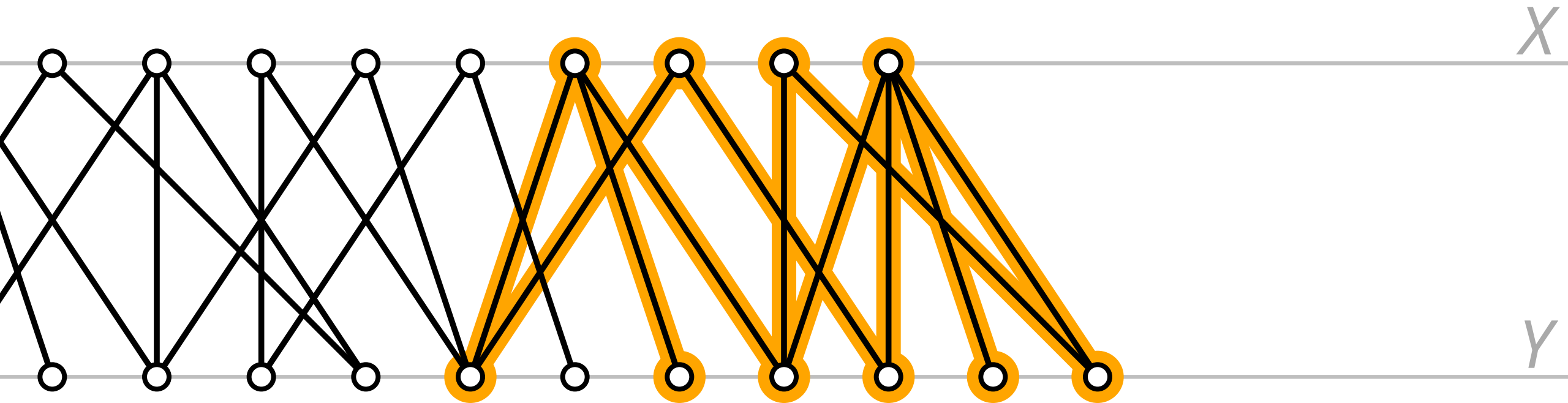
- Use the XP algorithm for 2-LAYER k -PLANARITY, which runs in $2^{O(k^3)} n^{4k+O(1)}$ time [KOW, SoCG 2025]
- Incorporate fan-planarity into the XP algorithm

Incorporating Fan-Planarity

The XP algorithm slides a "**window**" :

drawing of last $4k + 2$ vertices in X and the neighbors.

Tries to construct a 2-layer k -planar drawing with DP.



What we do

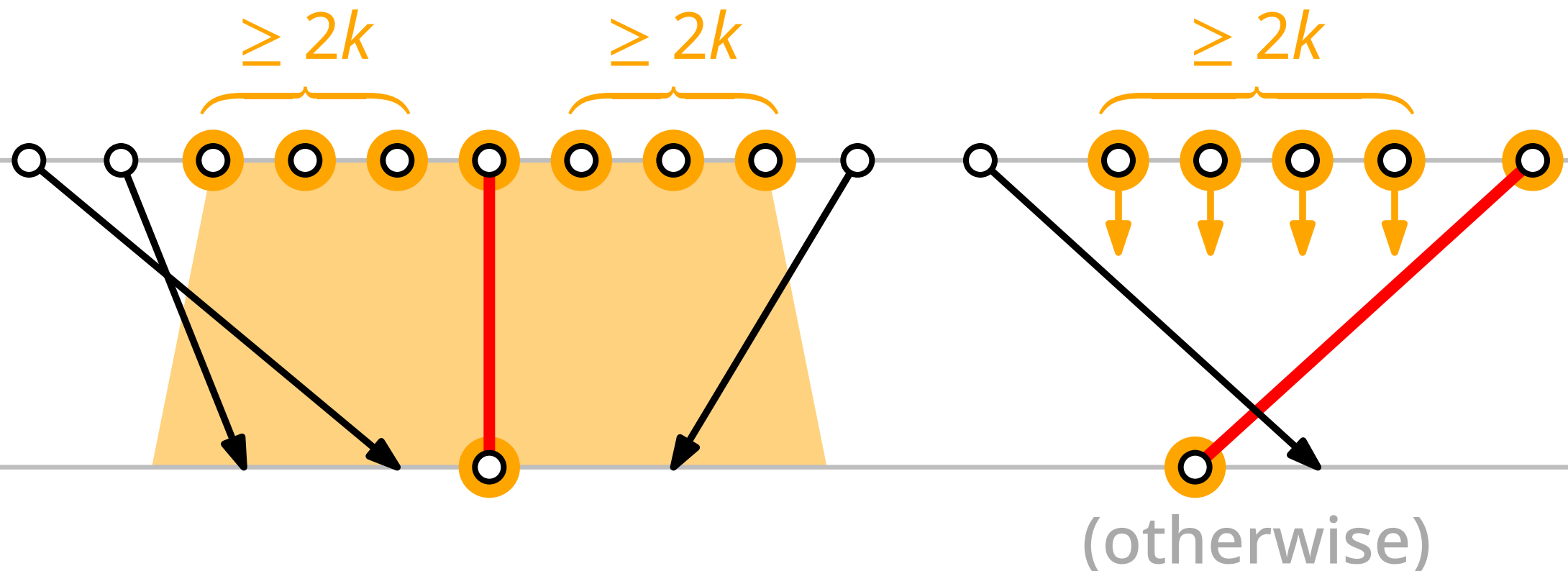
- require every "**window**" to be fan-planar.

Correctness

The modified DP returns Yes $\Leftrightarrow$
 $\exists$ 2-layer k -planar drawing with fan-planar windows.

Lemma Such a drawing is fan-planar.

Every edge has a "window" with all crossing edges.



Conclusions

Summary

- **Poly-time algorithm** for 2-LAYER FAN-PLANARITY
 - ① reduce degree to $\Delta(G) \leq 13$
 - ② add fan-planarity to 2-LAYER 13-PLANARITY

Future Work

- **Faster** (practical) polynomial-time algorithm?
- **One-Sided** version?
 - the same idea should work. (in the journal ver?)
- **Outer** (Circular) Fan-Planarity?
 - maximal: $O(n)$ time [BCGHK, GD 2014, Algorithmica]
- applicable to **other 2-layer beyond-planar** graphs?
 - for instance, 2-layer k -matching-planar?